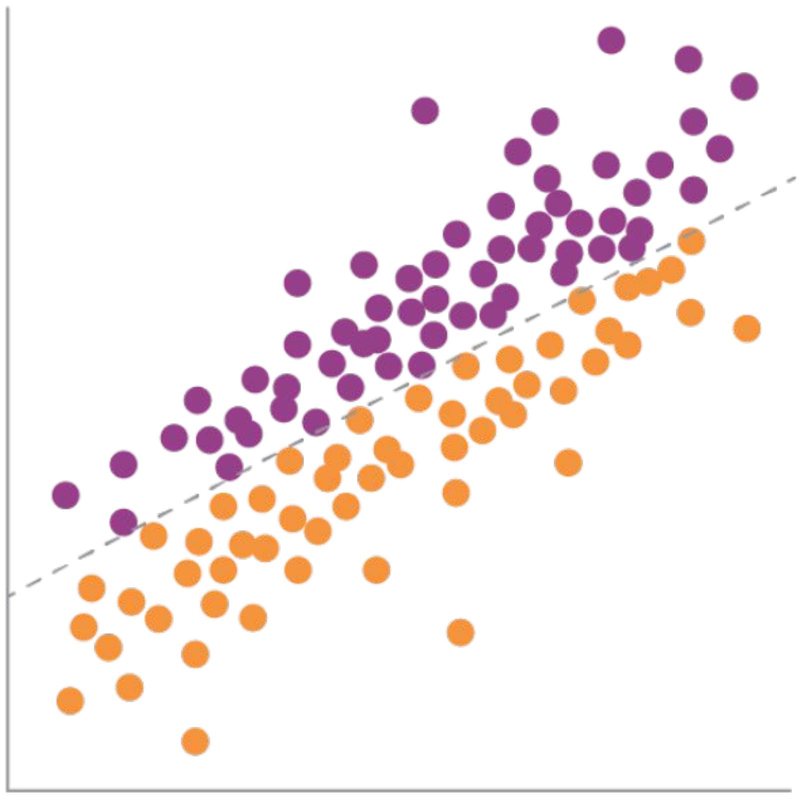
ENGR 3321: Introduction to Deep Learning for Robotics

Binary Classification

09/22/2025

Binary Classification



Review: Model Training

1. Load dataset: X (features), y (labels)
2. (Randomly) Initialize model parameters: w, b.
3. Evaluate the model with a metric (e.g. BCE).
4. Calculate gradient of loss.
5. Update parameters a small step on the directions descending the gradient of loss.
6. Repeat 3 to 5 until converge.

Load Dataset

A dataset with M samples:

- Each sample has N features: $x_1, x_2, \ldots, x_N$
- Each sample is labeled: y

$$\mathcal{D} = \{({}^{(1)}x_1, {}^{(1)}x_2, \ldots, {}^{(1)}x_N, {}^{(1)}y), ({}^{(2)}x_1, {}^{(2)}x_2, \ldots, {}^{(2)}x_N, {}^{(2)}y), \ldots, ({}^{(M)}x_1, {}^{(M)}x_2, \ldots, {}^{(M)}x_N, {}^{(M)}y)\}$$

$$= \{({}^{(1)}\mathbf{x}, {}^{(1)}y), ({}^{(2)}\mathbf{x}, {}^{(2)}y), \ldots, ({}^{(M)}\mathbf{x}, {}^{(M)}y)\}$$

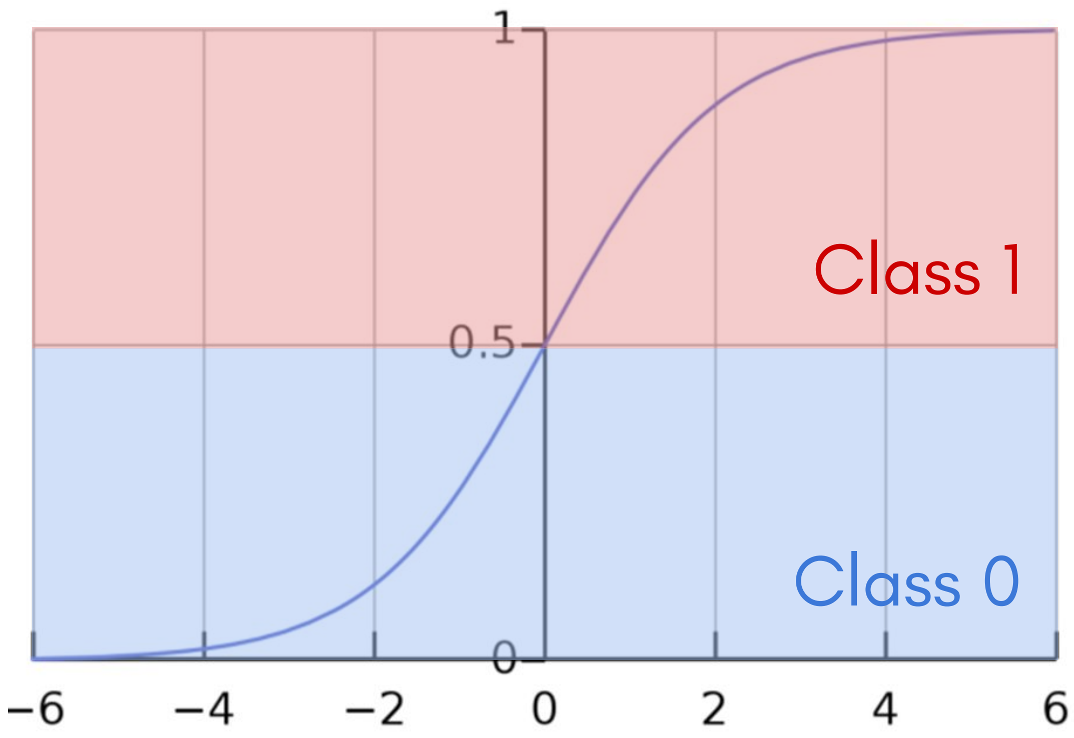
$${}^{(i)}y \in \{0, 1\}$$

Initialize Model

$$\hat{\mathbf{y}} = \sigma(\mathbf{X} \cdot \mathbf{w}^T + \mathbf{b}) = \sigma(\mathbf{z})$$

$(M, 1)$ (M, N) $(N, 1)$ $(M, 1)$ $(M, 1)$

Sigmoid Classification



Binary Cross Entropy Loss

$$\mathcal{L}(\hat{\mathbf{y}}, \mathbf{y}) = \frac{1}{M} \sum_{i=1}^{M} -^{(i)}y \ln {}^{(i)}\hat{y} - (1 - {}^{(i)}y) \ln(1 - {}^{(i)}\hat{y}) = \overline{-\mathbf{y} \ln \hat{\mathbf{y}} - (1 - \mathbf{y}) \ln(1 - \hat{\mathbf{y}})}$$

Binary Classification Metrics

PREDICTED \ ACTUAL	Positive	Negative
Positive	True Positive (TP) 5	False Positive (FP) 10
Negative	False Negative (FN) 15	True Negative (TN) 70

$$\text{Accuracy} = \frac{TP + TN}{TP + TN + FP + FN} = 0.75$$

$$\text{Recall} = \frac{TP}{TP + FN} = 0.25$$

E.g. airport security

$$\text{Precision} = \frac{TP}{TP + FP} = 0.33$$

E.g. investor

$$\text{F1 Score} = \frac{2 * \text{Precision} * \text{Recall}}{\text{Precision} + \text{Recall}} = 0.28$$

E.g. medical provider

The higher the better!

Gradient of Loss (BCE)

$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \left[\frac{\partial \mathcal{L}}{\partial w_1} \quad \frac{\partial \mathcal{L}}{\partial w_2} \quad \cdots \quad \frac{\partial \mathcal{L}}{\partial w_N}\right] = \frac{1}{M}(\hat{\mathbf{y}} - \mathbf{y})^T \cdot \mathbf{X}$$

$(1, N)$ $(1, M)$ (M, N)

Matrix Multiplication

$$\frac{\partial \mathcal{L}}{\partial b} = \overline{\hat{\mathbf{y}} - \mathbf{y}}$$

Gradient of Loss (MSE) Not Recommend

$$\frac{\partial \mathcal{L}}{\partial \mathbf{w}} = \left[\frac{\partial \mathcal{L}}{\partial w_1} \quad \frac{\partial \mathcal{L}}{\partial w_2} \quad \cdots \quad \frac{\partial \mathcal{L}}{\partial w_N}\right] = \frac{1}{M}[(\hat{\mathbf{y}} - \mathbf{y}) * \hat{\mathbf{y}} * (1 - \hat{\mathbf{y}})]^T \cdot \mathbf{X}$$

$$\frac{\partial \mathcal{L}}{\partial b} = \overline{(\hat{\mathbf{y}} - \mathbf{y}) * \hat{\mathbf{y}} * (1 - \hat{\mathbf{y}})}$$

Gradient Descent

Given dataset: $\left\{ \left({}^{(1)}\mathbf{x}, {}^{(1)}y \right), \left({}^{(2)}\mathbf{x}, {}^{(2)}y \right), \ldots, \left({}^{(M)}\mathbf{x}, {}^{(M)}y \right) \right\}$

Initialize $\mathbf{w}\, and\, b$

Repeat until converge {

$$\mathbf{w} := \mathbf{w} - \alpha \frac{\partial \mathcal{L}}{\partial \mathbf{w}}$$

$$b := b - \alpha \frac{\partial \mathcal{L}}{\partial b}$$

}

where α is learning rate