ENGR 3421: Robotics I

Kinematics of Differential Drive

10/14/2025

Outline

- Motion: From Motor to Robot
- Forward Kinematics

Encoder to Motor Velocity (Before Gearbox)

1. Encoder Counts Changing Rate = Encoder Counts Difference / Time Difference (s)

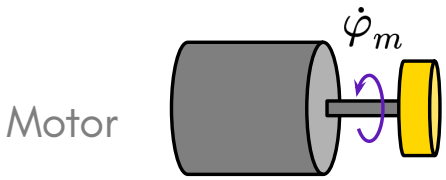
$$\dot{C} = \frac{\Delta C}{\Delta t} = \frac{C_t - C_{t-1}}{\Delta t}, \text{ counts/s}$$

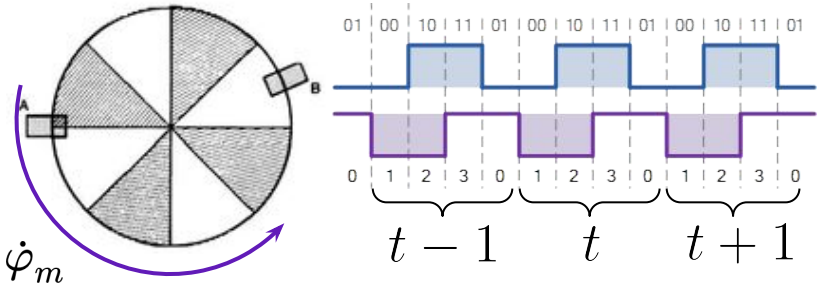
2. Angular Velocity = Encoder Counts Changing Rate / Counts Per Revolution

$$\dot{\tilde{\varphi}}_m = \frac{\dot{C}}{CPR}, \text{ revs/s}$$

3. Angular Velocity (rads/s) = 2 * pi (radians) * Angular Velocity (revs/s)

$$\dot{\varphi}_m = 2\pi\dot{\tilde{\varphi}}_m, \text{ rads/s}$$





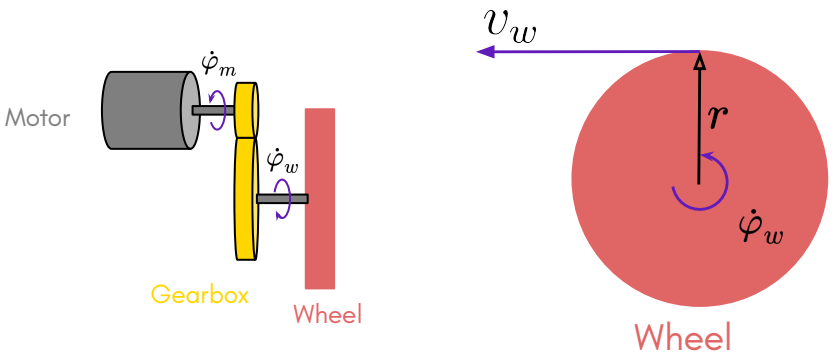
Motor to Wheel Velocity (After Gearbox)

1. Wheel's Angular Velocity (rads/s) = Motor's Angular Velocity / Gear Ratio

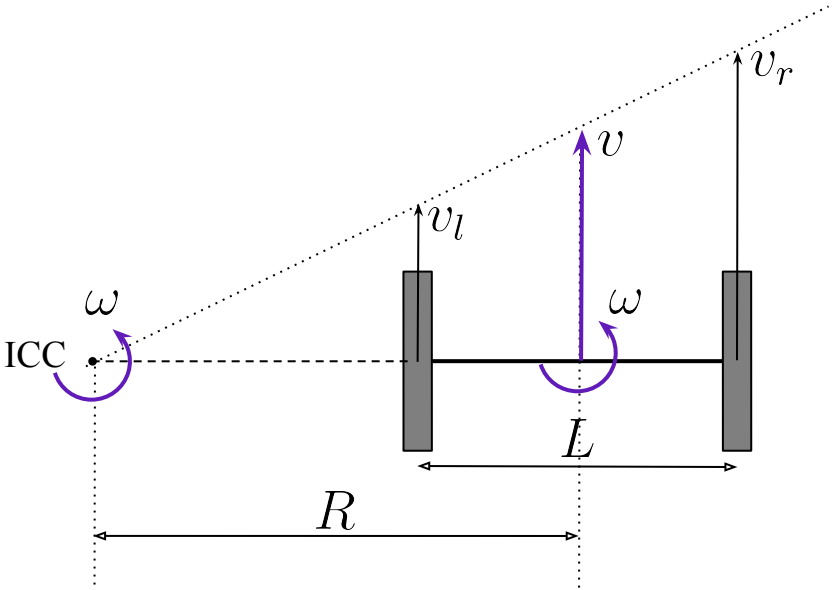
$$\dot{\varphi}_w = \frac{\dot{\varphi}_m}{i}, \text{ rads/s}$$

2. Wheel's Linear Velocity (m/s) = Wheel's Angular Velocity * Wheel's Radius (m)

$$v_w = \dot{\varphi}_w r, \text{ m/s}$$

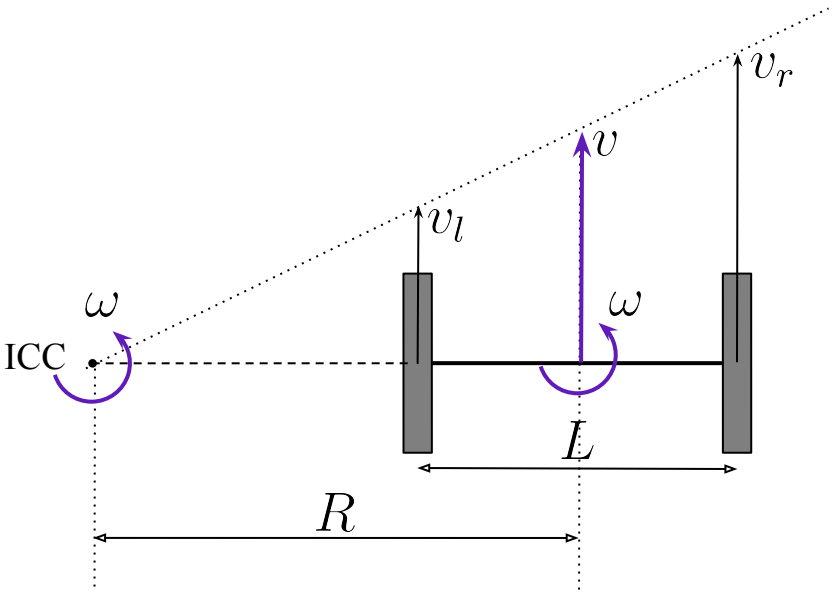


Motion Notations



ICC: Instantaneous Center of Curvature
R: radius of curvature
L: wheel separation distance
v: robot linear velocity
ω: robot angular velocity
v_l: linear velocity of left wheel
v_r: linear velocity of right wheel

Motion: From Wheel to Robot



$$\omega(R - \frac{L}{2}) = v_l$$
$$\omega(R + \frac{L}{2}) = v_r$$

Rotation about ICC must be same for both wheels.

$$v_l = v - \frac{\omega L}{2}$$

Linear velocity of left wheel

$$v_r = v + \frac{\omega L}{2}$$

Linear velocity of right wheel

$$R = \frac{L}{2}\frac{v_l + v_r}{v_l - v_r}$$

Rotation radius.

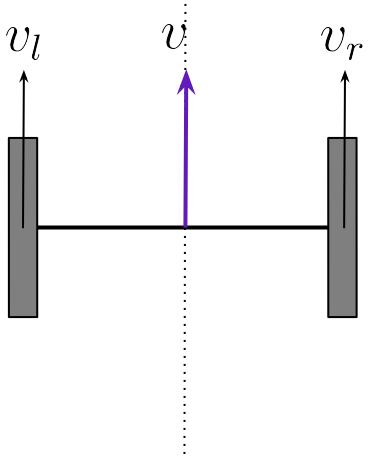
$$v = \frac{v_l + v_r}{2}$$

Robot's linear velocity

$$\omega = \frac{v_r - v_l}{L}$$

Robot's angular velocity

Special Case: $v_l = v_r$

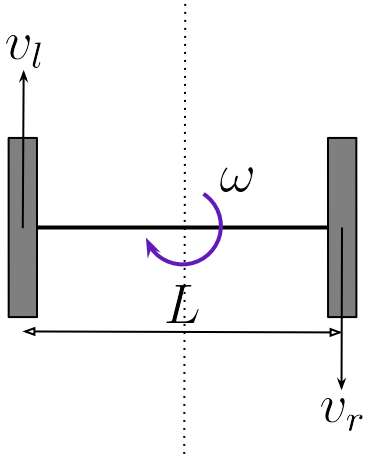


$$v = v_l = v_r$$

$$\omega = 0$$

ICC disappears.

Special Case: $v_l = -v_r$

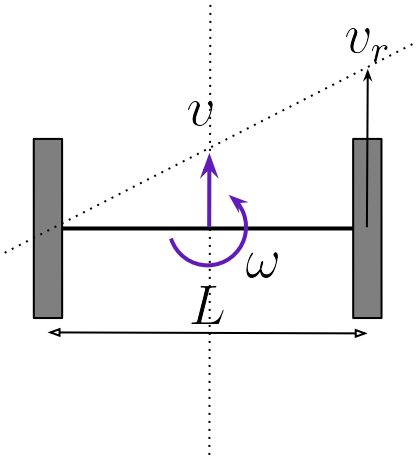


$$v = 0$$

$$\omega = \frac{2v_r}{L}$$

ICC at axle center.

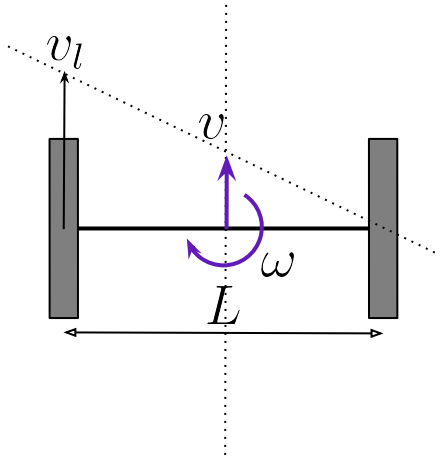
Special Case: $v_l = 0$ or $v_r = 0$



$$v = \frac{v_r}{2}$$

$$\omega = \frac{v_r}{L}$$

ICC at left wheel center.

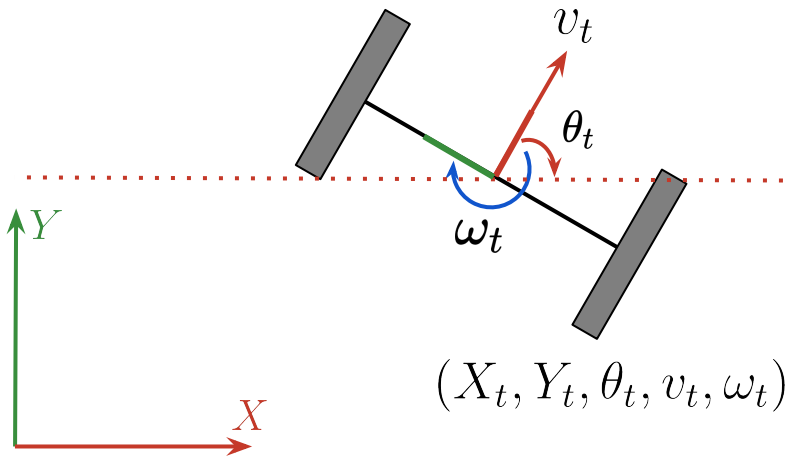


$$v = \frac{v_l}{2}$$

$$\omega = \frac{-v_l}{L}$$

ICC at right wheel center.

Forward Kinematics (Discrete)

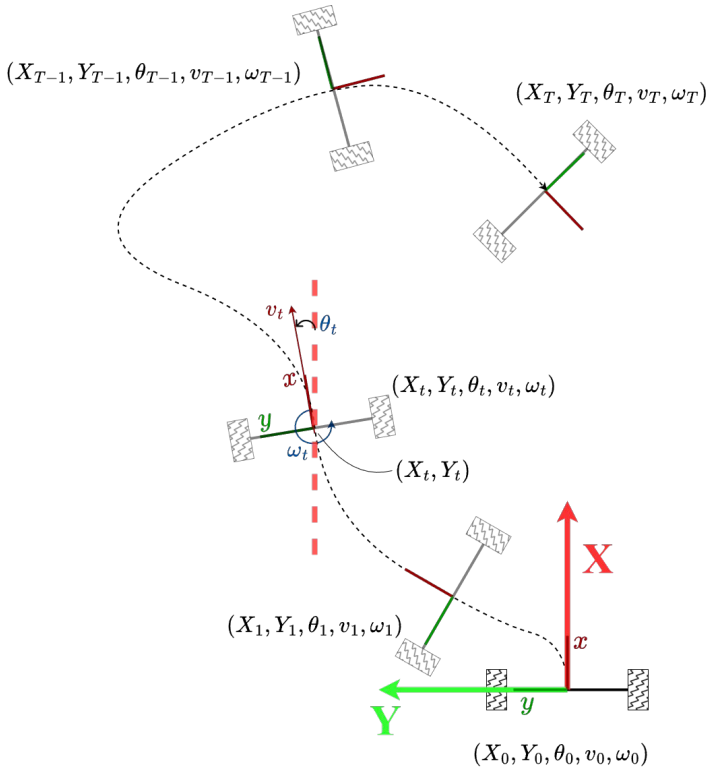


$$X_{t+1} = X_t + \Delta X$$
$$= X_t + v_t \cos\theta_t \Delta t$$

$$Y_{t+1} = Y_t + \Delta Y$$
$$= Y_t + v_t \sin\theta_t \Delta t$$

$$\theta_{t+1} = \theta_t + \Delta\theta$$
$$= \theta_t + \omega_t \Delta t$$

Forward Kinematics (Discrete)



$$
\begin{aligned}
X_{t+1} &= X_t + \Delta X \\
&= X_t + v_t \cos\theta_t \Delta t
\end{aligned}
$$

$$
\begin{aligned}
Y_{t+1} &= Y_t + \Delta Y \\
&= Y_t + v_t \sin\theta_t \Delta t
\end{aligned}
$$

$$
\begin{aligned}
\theta_{t+1} &= \theta_t + \Delta\theta \\
&= \theta_t + \omega_t \Delta t
\end{aligned}
$$