ENGR 3421: Robotics I

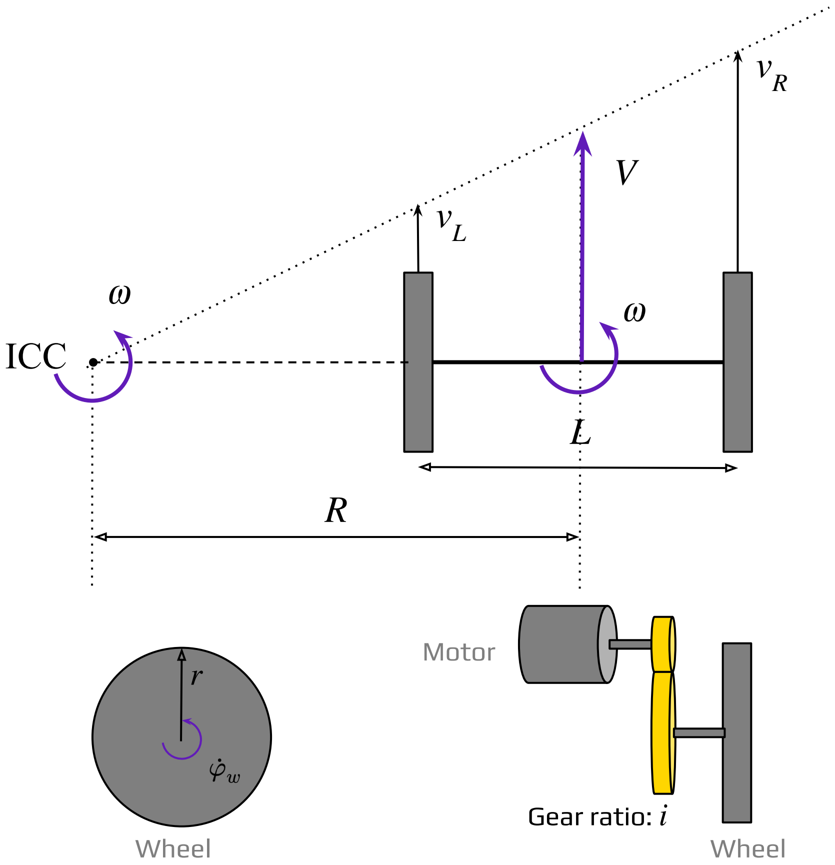
Kinematics of Differential Drive

10/17/2023

Outline

- Motion: From Motor to Robot
- Forward Kinematics (w.r.t. different frames)

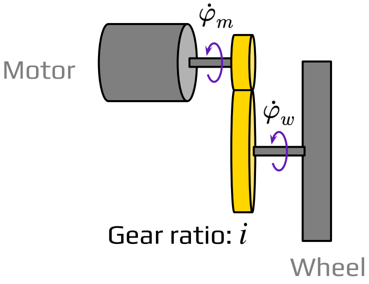
Motion: From Motor to Robot

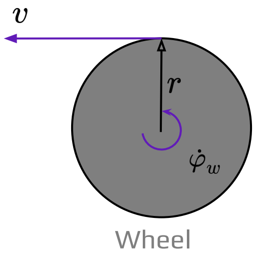


ICC: Instantaneous Center of Curvature
R: radius of curvature
L: wheel separation distance
V: robot linear velocity
ω: robot angular velocity
r: radius of wheel
i: gear ratio
$\dot{\varphi}_w$: angular velocity of wheel
$\dot{\varphi}_m$: angular velocity of motor
v_L: linear velocity of left wheel
v_R: linear velocity of right wheel

Speed Computation: Motor to Wheel

1. Time "Counts Per Second"
2. Revolutions Per Second = Counts Per Second / Counts Per Revolution
3. Shaft Speed = Revolutions Per Second / Gear Ratio = Wheel Angular Speed
4. Wheel Linear Speed = Wheel Angular Speed * Wheel Radius





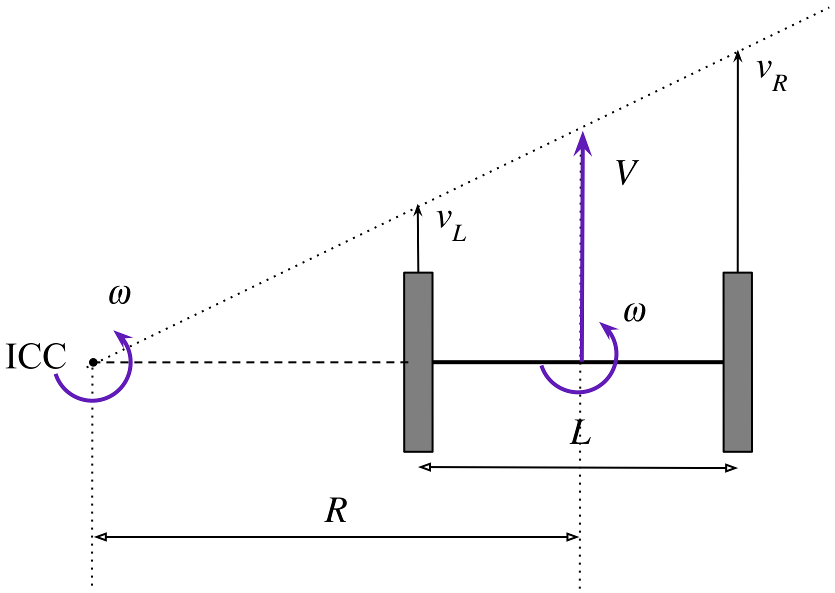
$$\dot{\varphi}_w = \frac{\dot{\varphi}_m}{i}$$

Motor velocity to wheel velocity

$$v = \dot{\varphi}_w r$$

Wheel angular velocity to linear velocity

Motion: From Wheel to Robot



$$\omega\left(R - \frac{L}{2}\right) = v_L$$

$$\omega\left(R + \frac{L}{2}\right) = v_R$$

Rotation about ICC must be same for both wheels.

$$v_L = V - \frac{\omega L}{2}$$

Linear velocity of left wheel

$$v_R = V + \frac{\omega L}{2}$$

Linear velocity of right wheel

$$V = \frac{v_L + v_R}{2}$$

Linear velocity of robot

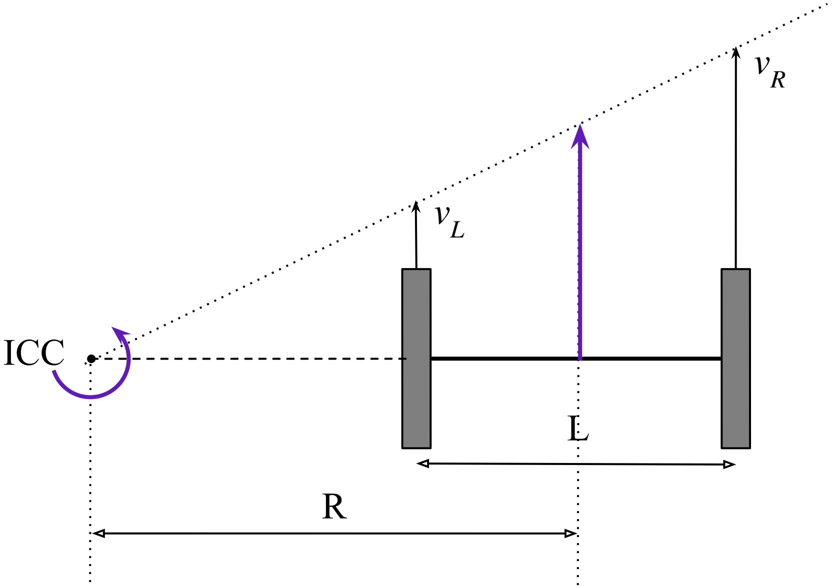
$$\omega = \frac{v_R - v_L}{L}$$

Angular velocity of robot

$$R = \frac{L}{2} \frac{v_L + v_R}{v_L - v_R}$$

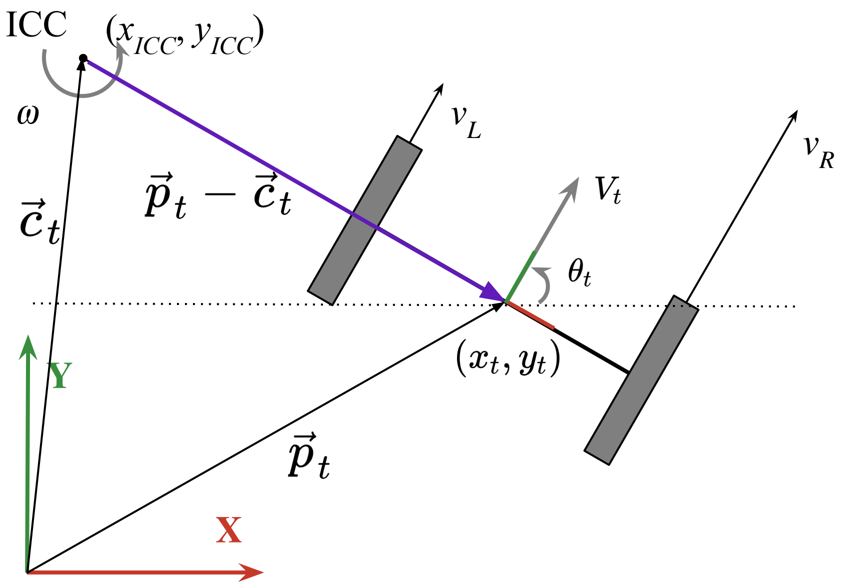
Rotation radius.

Motion: Special Cases



- If $v_L = v_R$, then linear motion in a straight line. R becomes infinite, no rotation ω=0.
- If $v_L = -v_R$, then rotation about the midpoint of the wheel axis, R = 0.
- If $v_L = 0$, then rotation about the left wheel, R = L/2. Rotation about the right wheel if $v_R = 0$.

Forward Kinematics (Continuous)



$$\vec{p} = [x,\, y]^T$$

$$\vec{c} = [x - R\sin(\theta),\, y + R\cos(\theta)]^T$$

$$\vec{p} - \vec{c} = [x - x_{ICC},\, y - y_{ICC}]^T$$

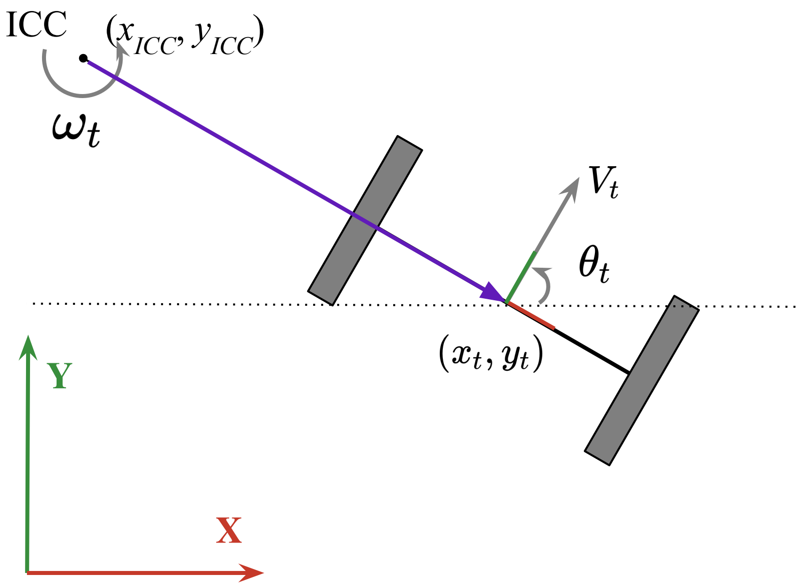
$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega\delta t) & -\sin(\omega\delta t) & 0 \\ \sin(\omega\delta t) & \cos(\omega\delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - x_{ICC} \\ y - y_{ICC} \\ \theta \end{bmatrix} + \begin{bmatrix} x_{ICC} \\ y_{ICC} \\ \omega\delta t \end{bmatrix}$$

$$x(t) = \int_0^t V(t)\cos(\theta(t))dt$$

$$y(t) = \int_0^t V(t) sin(\theta(t))dt$$

$$\theta(t) = \int_0^t \omega(t)dt$$

Forward Kinematics (Discrete)

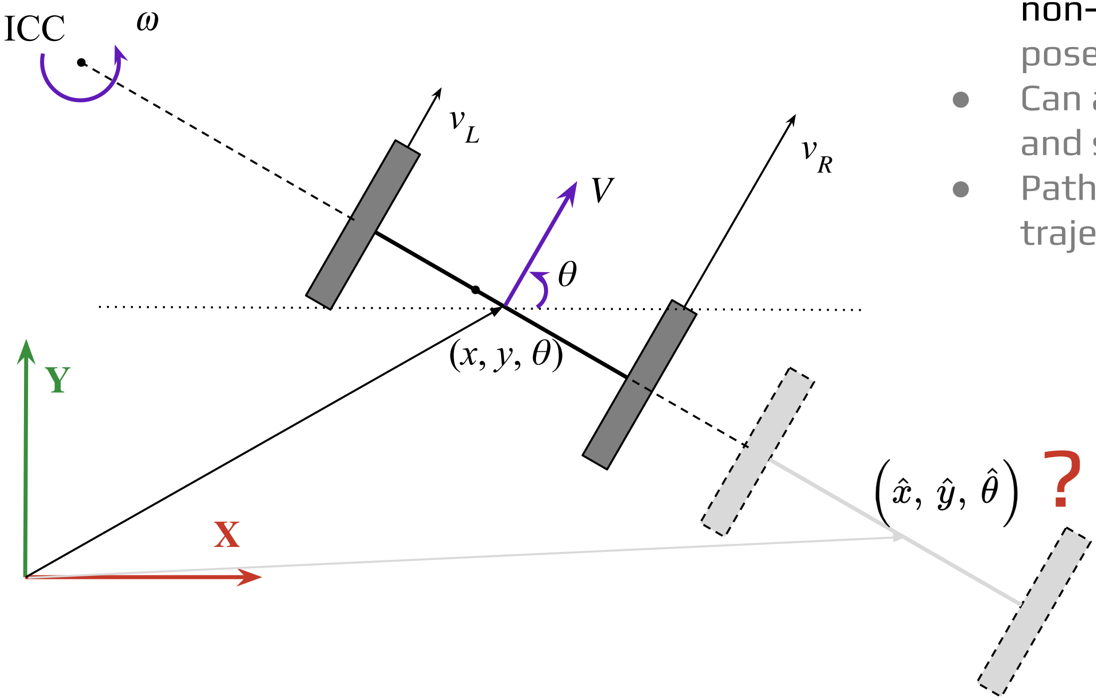


$$x_{t+1} = x_t + \Delta x$$
$$= x_t + V_t \cos\theta_t \cdot \Delta t$$

$$y_{t+1} = y_t + \Delta y$$
$$= y_t + V_t sin\theta_t \cdot \Delta t$$

$$\theta_{t+1} = \theta_t + \Delta\theta$$
$$= \theta_t + \omega_t \cdot \Delta t$$

Inverse Kinematics



- Given a target $(\hat{x}, \hat{y}, \hat{\theta})$, What is $V(t)$ and $\omega(t)$?
- Two-wheeled differential drive vehicle imposes non-holonomic constraints on establishing its pose (think about lateral translation).
- Can achieve the goal by moving in straight line and spinning in place.
- Path planning algorithms may find smoother trajectories.